# MATHEMATICS SPECIALIST
# UNITS 1&2
**Section One:**
**Calculator-free**

Your Name_____

Your Teacher's Name_____

## Time allowed for this section
Reading time before commencing work:     five minutes
Working time:                            fifty minutes

## Materials required/recommended for this section
***To be provided by the supervisor***
This Question/Answer booklet
Formula sheet

***To be provided by the candidate***
Standard items:     pens (blue/black preferred), pencils (including coloured), sharpener,
                    correction fluid/tape, eraser, ruler, highlighters

Special items:     nil

## Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised material. If you have any unauthorised material with you,
hand it to the supervisor **before** reading any further.

| Question | Mark | Max | Question | Mark | Max |
|----------|------|-----|----------|------|-----|
| 1 | | 4 | 6 | | 5 |
| 2 | | 9 | 7 | | 7 |
| 3 | | 7 | 8 | | 5 |
| 4 | | 9 | | | |
| 5 | | 4 | | | |

## Structure of this paper

| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
|---------|------------------------------|-----------------------------------|-----------------------|-----------------|--------------------------|
| Section One: Calculator-free | 8 | 8 | 50 | 50 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 93 | 65 |
| | | | | **Total** | 100 |

## Instructions to candidates

1.  The rules for the conduct of the Western Australian Certificate of Education ATAR course examinations are detailed in the *Year 12 Information Handbook 2020*. Sitting this examination implies that you agree to abide by these rules.

2.  Write your answers in this Question/Answer booklet.

3.  You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4.  Additional pages for the use of planning your answer to a question or continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number.

5.  **Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6.  It is recommended that you **do not use pencil**, except in diagrams.

7.  The Formula sheet is **not** to be handed in with your Question/Answer booklet.

**Section One: Calculator-free**                                                        **(50 marks)**

This section has **8 (eight)** questions. Answer **all** questions. Write your answers in the spaces provided.

Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
*   Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
*   Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

Working time: 50 minutes.

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**Question 1 (2.3.1)**                                                                 **(4 marks)**

Prove the following statement:

If $m$ and $n$ are odd integers, then $m^2 - n^2$ is divisible by 4.

**Question 2 (2.2.1 - 2.2.3, 2.2.11)** **(9 marks)**

a) Let $M = \begin{bmatrix} 3 & 4 \\ 1 & -2 \end{bmatrix}$.

    i.      Determine $M^{-1}$.           (2 marks)

    ii.     Showing use of an appropriate matrix equation together with your answer to part (i), determine the coordinates of the point of intersection of the lines $3x + 4y = -1$ and $x - 2y = 8$.

                              (3 marks)

b) Consider the equation

$$\begin{bmatrix} 6 & 4 \\ k & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ 4 \end{bmatrix}$$

where $k \in \mathbb{R}$.

i.      Determine the value of $k$ such that the equation **does not** have a unique
        solution for $\begin{bmatrix} x \\ y \end{bmatrix}$.                                    (2 marks)

ii.     With the value of $k$ obtained in part (i), what is the geometrical relationship
        between the lines $6x + 4y = -8$ and $kx - 2y = 4$?                    (2 marks)

**Question 3 (2.3.4, 2.3.5)**                                                   **(7 marks)**

Use the principle of mathematical induction to prove that

$$4 + 32 + 108 + \cdots + 4n^3 = n^2(n+1)^2$$

for all integers $n \geq 1$.

**Question 4 (2.3.7-2.3.10)** **(9 marks)**

Let $z = 3 - 5i$ and $w = -2 + i$. Write each of the following in the form $a + bi$ where $a, b \in \mathbb{R}$.

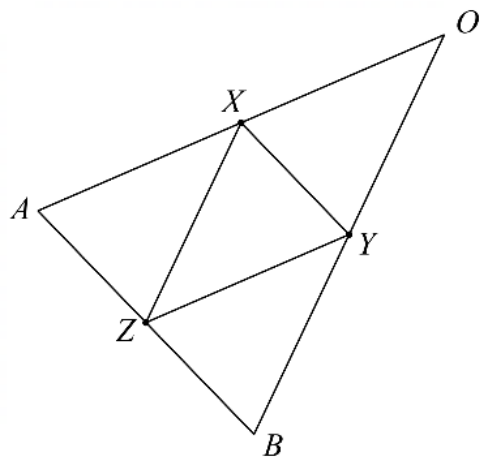a) $z + w$ (2 marks)

b) $3zw$ (2 marks)

c) $z + \bar{z}$ (2 marks)

d) $\dfrac{z}{w}$ (3 marks)

**Question 5 (1.2.11, 1.2.12)** **(4 marks)**

Let $\Delta OAB$ be an isosceles triangle with $OA = OB$, and let $X, Y$ and $Z$ be the midpoints of $\overline{OA}$, $\overline{OB}$ and $\overline{AB}$ respectively. Let $\boldsymbol{a} = \overrightarrow{OA}$ and $\boldsymbol{b} = \overrightarrow{OB}$.

Use a vector method to prove that $\Delta XYZ$ is isosceles.

**Question 6 (2.1.3 - 2.1.6, 2.1.8)** **(5 marks)**

Prove that the following is true for all $\theta$.

$$2 \sin \theta - \sin 2\theta \cos \theta = 2 \sin^3 \theta$$

**Question 7 (1.3.2, 1.3.5, 2.3.1)** **(7 marks)**

a) Let $p$ be an irrational number and $q$ a rational number. Use the method of proof by contradiction to prove that $pq$ is irrational. (4 marks)

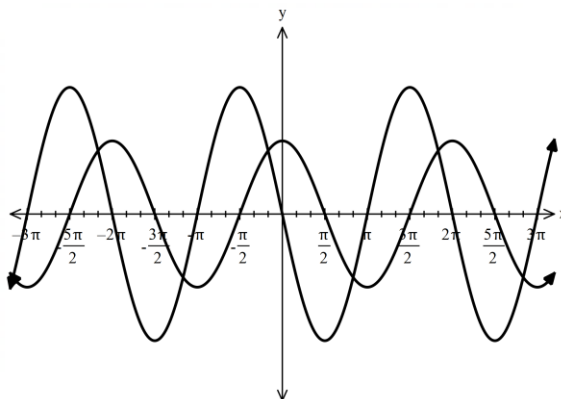b) State whether the following is true or false and prove or disprove it accordingly:
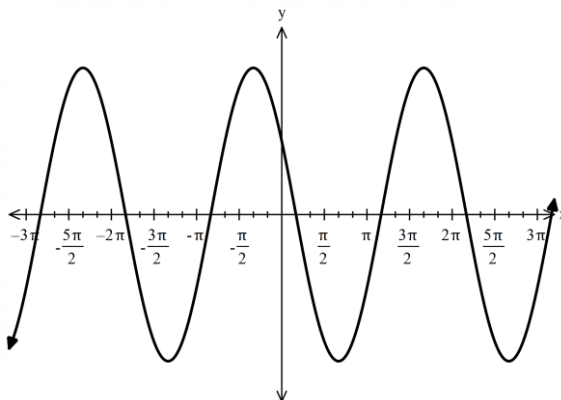   'If $p$ is irrational and $q$ is rational, then $(p + q)^2$ is irrational.'
   (3 marks)

**Question 8 (2.1.7)** **(5 marks)**

The diagram below shows the graphs of two functions $f(x) = 3\cos(x)$ and $g(x) = a\sin(x)$ where $a$ is a real constant.



Below is the graph of $y = f(x) + g(x)$, which can also be expressed as $y = b\sin(x + \alpha)$ where $b$ and $\alpha$ are positive real constants.



a) Determine an exact value for $\alpha$. (1 mark)

b) Determine the values of $a$ and $b$. (4 marks)

**END OF SECTION ONE**

**Additional working space**

Question number:

**Additional working space**

Question number:

**Additional working space**

Question number:

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Question number: